Common Derivatives

Fangyuan Lin https://fangyuanlin2002.github.io/

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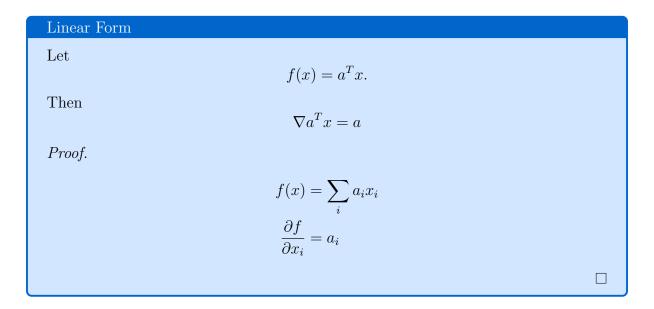
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0.1 Remarks

- Find it annoying to always forget the common multivariate derivatives? Me too.
- All variables are assumed to be vectors of appropriate dimensions and \vec{x} is written as x for convenience.

0.2 Common Derivatives

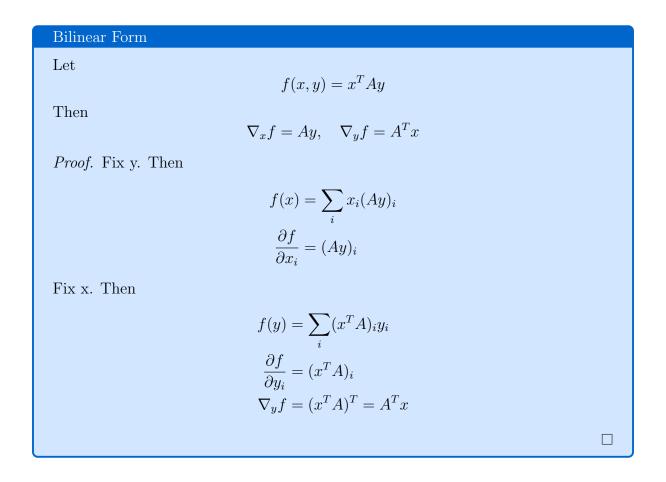
0.2.1 Linear Form



0.2.2 Affine Function

Affine Function f(x) = Ax + b Then $J_f(x) = A$ Proof. $f_i(x) = \sum_j A_{ij}x_j + b_i$ $\frac{\partial f_i}{\partial x_j} = A_{ij}$

0.2.3 Bilinear Form / Inner Product



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0.2.4 Quadratic Form

Quadratic Form

Let

$$f(x) = x^T A x$$

Then

$$\nabla f = (A + A^T)x$$

$$f(x) = \sum_{i} x_{i} (Ax)_{i}$$

$$= \sum_{i} x_{i} (\sum_{j} A_{ij} x_{j})$$

$$= \sum_{i} \sum_{j} x_{i} A_{ij} x_{j}$$

$$\frac{\partial f}{\partial x_{k}} = \sum_{j \neq k} A_{kj} x_{j} + 2A_{kk} x_{k} + \sum_{i \neq k} A_{ik} x_{i}$$

$$= (Ax)_{k} + (A^{T}x)_{k}$$

0.2.5 Squared Norm

Squared Norm

Let

$$f(x) = ||x||^2 = x^T x.$$

Then

$$\nabla f(x) = 2x$$

Proof. Note that his is a special case of quadratic form with A=I. Also one can consider:

$$f(x) = \sum_{i} x_i^2$$

$$\frac{\partial f}{\partial x_i} = 2x_i$$